



Competition #3

The Junior Online Math Olympiad

31st March 2014 - 7th April 2014

Short Questions

1. If $8^m = 27$, so what is the value of 4^m ?
2. If I divide 20 by 14, what are the first three digits of the sum of the first 2014 digits after the decimal point?
3. In English Class, after studying a book called “The Lord of the Flies” by William Golding, students are split into 6 groups and each group is required to make a film based on one of the 8 important scenes in the novel.

Each group can choose which scene they are creating a film of. However, to make sure there is a variety of scenes being created, there can only be a maximum of 2 groups creating the same scene.

Find the last 3 digits of the number of possible ways there are for each group to choose a scene.

Details and Assumptions

Each group has different students in it, so therefore each group is considered distinct.

4. Given that:

$$N = 0.10110101\overline{01}$$

and N can be expressed as $\frac{a}{b}$ where a and b are positive coprime integers. What is the value of the digit sum of $a + b$?

5. The number of ways there are to arrange 6 distinct rooks on a 10×10 board such that none of them can attack each other is n . Find the product of the non-zero digits of n

Details and Assumptions

A rook is a chess piece that can only move vertically and horizontally across the board. Therefore, the rooks have to be in different rows and columns of the board as each other.

6. Find the largest n such that $100^n | 100!$

Details and Assumptions

- $a|b$ means that a is a factor of b
 - $n!$ is the factorial function
7. If the expression $(x^{10} + 5x - 1)^8$ is completely expanded, what is the last three digits of the sum of the coefficients of the terms with even powers of x ?
8. 4 identical circles of radius r is inscribed in a circle with radius 1 and are tangent to each other. Find the value of $r(r + 2)$
9. A circle of radius 1 is inscribed in a square which is in turn inscribed in a bigger circle which is in turn inscribed in a bigger square. Find the area of the larger square.
10. If the sum of the solutions $x \in [0^\circ, 270^\circ]$ to:

$$\cos 2x + 2 \sin x - \sqrt{3} \sin x = 1 - \sqrt{3}$$

is S° , then find S .

Long Questions

Explain your answer for each of the questions

1. Let ρ be a root of: $x^2 - 2x - 1 = 0$

Define a function $F(n) | n \in \mathbb{N}$ by the recurrence relation:

- $F(0) = 0$
- $F(1) = 1$
- $F(n) = 2F(n - 1) + F(n - 2) | n \geq 2$

Prove that for all integers $n \geq 1$:

$$\rho^n = F(n - 1) + F(n)\rho$$

(3 points)

2. Let the roots of the function: $f(x) = x^3 - 5x^2 + 6x + 1$ be α , β and γ

Find, with proof, the value of $(\alpha + 1)(\beta + 1)(\gamma + 1)$

(3 Points)

3. Show that

$$\sin^4(x) - \cos^4(x) = \sin^2(x) - \cos^2(x)$$

(2 points)